РАСЧЕТНО - ГРАФИЧЕСКАЯ РАБОТА
ПО КУРСУ “ТЕОРИЯ УПРАВЛЕНИЯ”

1. РГР состоит из 2 частей, соответствующих разделам курса "Вариационное исчисление" и "Оптимальное управление". Каждая часть оформляется в отдельной тетради или на листах А4 (обязательно пронумерованных в порядке следования заданий)
2. Образец оформления обложки:

Расчетно-графическая работа
по курсу "Теория управления"
(часть 1)
студента III курса
специальности "Прикладная математика"
Петрова Ивана

ЧАСТЬ 1. ВАРИАЦИОННОЕ ИСЧИСЛЕНИЕ (ПРЕДВАРИТЕЛЬНЫЙ СРОК СДАЧИ 25 АПРЕЛЯ 2018 Г.)

1. (3 б.) Найти экстремали функционала:

1. $v[y]=\int\limits_1^e\left(\frac{2y}{x}+yy'+x^2(y')^2\right)dx,\quad y(1)=1,\quad y(e)=0;$

2. $v[y]=\int\limits_1^2\left(2y+yy'+x^2(y')^2\right)dx,\quad y(1)=0,\quad y(2)=1+\ln 2;$

3. $v[y]=\int\limits_0^{\pi/4}\left(4y^2+(y')^2+8y\right)dx,\quad y(0)=-1,\quad y\left(\frac{\pi}{4}\right)=0;$

4. $v[y]=\int\limits_1^2\left(\frac{3y^2}{x^3}+x^2+\frac{(y')^2}{x}\right)dx,\quad y(1)=2,\quad y(2)=8\frac{1}{2};$

5. $v[y]=\int\limits_0^{\pi/2}\left((y')^2+4y^2+2y\cos x\right)dx,\quad y(0)=\frac{4}{5},\quad y\left(\frac{\pi}{2}\right)=e^{\pi};$

6. $v[y]=\int\limits_{-2}^{-1}\left(x^2(y')^2+12y^2\right)dx,\quad y(-2)=\frac{1}{16},\quad y(-1)=1;$

7. $v[y]=\int\limits_1^3\left(2y-yy'+x(y')^2\right)dx,\quad y(1)=1,\quad y(3)=4;$

8. $v[y]=\int\limits_0^{\pi}\left((y'+y)^2+2y\sin x\right)dx,\quad y(0)=0,\quad y(\pi)=1;$

9. $v[y]=\int\limits_0^1\left((y')^2+y^2+2e^{2x}y\right)dx,\quad y(0)=\frac{1}{3},\quad y(1)=\frac{1}{3}e^2;$

10. $v[y]=\int\limits_1^2\left(\frac{3y^2}{x^3}+\frac{(y')^2}{x}+8y\right)dx,\quad y(1)=0,\quad y(2)=8\ln 2.$

2. (2 б.) Найти экстремаль функционала, удовлетворяющую заданным граничным условиям:

1. $v[y]=\int\limits_0^3\frac{y'}{\sqrt{1+(y')^2}}dx,\quad y(0)=1,\quad y(3)=4;$

2. $v[y]=\int\limits_1^2\frac{x^2(y')^2}{2x^3+1}dx,\quad y(1)=0,\quad y(2)=\frac{7}{2};$

3. $v[y]=\int\limits_2^7\left(\cos x+3x^2y+(x^3-y^2)y'\right)dx,\quad y(2)=3,\quad y(7)=0;$

4. $v[y]=\int\limits_0^2\left(6y'^2-(y')^4+yy'\right)dx,\quad y(0)=0,\quad y(2)=3;$

5. $v[y]=\int\limits_0^2\left(xy'+(y')^2\right)dx,\quad y(0)=1,\quad y(2)=0;$

6. $v[y]=\int\limits_0^1\left(6x^2y'+(y')^2\right)dx,\quad y(0)=0,\quad y(1)=-1;$

7. $v[y]=\int\limits_1^2\left(yx^2-y+xy^2y'\right)dx,\quad y(1)=0,\quad y(2)=\sqrt{3};$

8. $v[y]=\int\limits_0^{\frac{\pi}{4}}\left(4y^2-(y')^2+8y\right)dx,\quad y(0)=-1,\quad y\left(\frac{\pi}{4}\right)=0;$

9. $v[y]=\int\limits_1^5\frac{12(y')^3+7(y')^4}{(y')^6+5}dx,\quad y(1)=2,\quad y(5)=14;$

10. $v[y]=\int\limits_1^2\frac{\sqrt{1+(y')^2}}{(y')^3}dx,\quad y(1)=-3,\quad y(2)=-8.$

3. (3 б.) Найти экстремали функционалов:

1. $v[y,z]=\int\limits_0^{\pi}(2yz-2y^2+(y')^2-(z')^2)dx,\quad y(0)=0,\ y(\pi)=1,\ z(0)=0,\ z(\pi)=-1;$

2. $v[y,z]=\int\limits_0^{\frac{\pi}{2}}(2yz+(y')^2+(z')^2)dx,\quad y(0)=1,\ y\left(\frac{\pi}{2}\right)=e^{\frac{\pi}{2}},\ z(0)=1,\ z\left(\frac{\pi}{2}\right)=e^{\frac{\pi}{2}};$

3. $v[y,z]=\int\limits_0^{\frac{\pi}{4}}(2z-4y^2+(y')^2-(z')^2)dx,\quad y(0)=0,\ y\left(\frac{\pi}{4}\right)=1,\ z(0)=0,\ z\left(\frac{\pi}{4}\right)=1;$

4. $v[y,z]=\int\limits_0^{\frac{\pi}{4}}(2y-4z^2+(z')^2-(y')^2)dx,\quad y(0)=0,\ y\left(\frac{\pi}{4}\right)=1,\ z(0)=0,\ z\left(\frac{\pi}{4}\right)=1;$

5. $v[y,z]=\int\limits_0^{\frac{\pi}{2}}((y')^2+(z')^2-2yz)dx,\quad y(0)=0,\ y\left(\frac{\pi}{2}\right)=1,\ z(0)=0,\ z\left(\frac{\pi}{2}\right)=1;$

6. $v[y,z]=\int\limits_0^1((y')^2+(z')^2+2y)dx,\quad y(0)=1,\ y(1)=\frac{3}{2},\ z(0)=0,\ z(1)=1;$

7. $v[y,z]=\int\limits_0^1(2y+z^2+(y')^2+(z')^2)dx,\quad y(0)=0,\ y(1)=\frac{1}{2},\ z(0)=1,\ z(1)=e^{-1};$

8. $v[y,z]=\int\limits_1^2(y'^2+z^2+(z')^2)dx,\quad y(1)=1,\ y(2)=2,\ z(1)=0,\ z(2)=1;$

9. $v[y,z]=\int\limits_0^1(2y^2+2yz+(y')^2-(z')^2)dx,\quad y(0)=0,\ y(1)=2sh1,\ z(0)=0,\ z(1)=-2sh1;$

10. $v[y,z]=\int\limits_{-1}^1\left(2xy-(y')^2+\frac{1}{3}(z')^3\right)dx,\quad y(-1)=2,\ y(1)=0,\ z(-1)=-1,\ z(1)=1.$

4. (3 б.) Найти экстремали функционалов:

1. $v[y]=\int\limits_{-1}^0(240y-(y''')^2)dx,\quad y(-1)=1,\ y(0)=0,\ y'(-1)=-4.5,\ y'(0)=0,\ y''(-1)=16,\ y''(0)=0;$

2. $v[y]=\int\limits_0^1(y^2+2(y')^2+(y'')^2)dx,\quad y(0)=0,\ y(1)=0,\ y'(0)=1,\ y'(1)=-sh1;$

3. $v[y]=\int_0^{\frac{\pi}{2}}(2y\sin x+(y'')^2)dx,\quad y(0)=0,\ y\left(\frac{\pi}{2}\right)=-1,\ y'(0)=-1,\ y'\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4};$

4. $v[y]=\int_0^1(y^2-2yx+(y'')^2)dx,\quad y(0)=0,\ y(1)=1,\ y'(0)=1,\ y'(1)=1;$

5. $v[y]=\int_0^{\frac{\pi}{2}}((y'')^2-y^2+x^2)dx,\quad y(0)=1,\ y\left(\frac{\pi}{2}\right)=0,\ y'(0)=0,\ y'\left(\frac{\pi}{2}\right)=-1;$

6. $v[y]=\int_0^1(-2xy+(y'')^2)dx,\quad y(0)=0,\ y(1)=\frac{1}{5!},\ y'(0)=0,\ y'(1)=\frac{1}{12};$

7. $v[y]=\int_0^1(2e^xy-(y'')^2)dx,\quad y(0)=1,\ y(1)=e,\ y'(0)=1,\ y'(1)=2e;$

8. $v[y]=\int_0^{\frac{\pi}{2}}((y'')^2+y^2-2(y')^2)dx,\quad y(0)=0,\ y\left(\frac{\pi}{2}\right)=\frac{\pi}{2},\ y'(0)=0,\ y'\left(\frac{\pi}{2}\right)=1;$

9. $v[y]=\int_1^2((y''')^2+y^2-2yx^3)dx,\quad y(1)=1,\ y(2)=8,\ y'(1)=3,\ y'(2)=12,\ y''(1)=6,\ y''(2)=12;$

10. $v[y]=\int_0^{\frac{\pi}{4}}((y'')^2-4(y')^2)dx,\quad y(0)=0,\ y\left(\frac{\pi}{4}\right)=\frac{\pi}{2}-2,\ y'(0)=0,\ y'\left(\frac{\pi}{4}\right)=0.$

5. (3 б.) Найти экстремали в задаче с подвижным или свободным концом:

1. $v[y]=\int_0^{x_1}\frac{\sqrt{1+y'^2}}{y}dx,\quad y(0)=1,\quad y_1=x_1-1;$

2. $v[y]=\int_1^2\left(x^3y'^2+3xy^2\right)dx,\quad y(2)=\frac{49}{24};$

3. $v[y]=\int_0^{x_1}\frac{\sqrt{1+y'^2}}{x-3}dx,\quad y(0)=0,\quad y_1+4x_1-4=0;$

4. $v[y]=\int_1^3\left(8yy'\ln x-xy'^2+6xy'\right)dx,\quad y(3)=15;$

5. $v[y]=\int_1^2\left(x^2y'^2+12y^2\right)dx,\quad y(1)=97;$

6. $v[y]=\int_{x_0}^{x_1}\sqrt{1+y'^2}dx,\quad y_0=x_0^2,\quad y_1=x_1-1;$

7. $v[y]=\int_1^2\left(x^2y'^2+6y^2+2x^3y\right)dx,\quad y(1)=\frac{1}{6};$

8. $v[y]=\int_1^2\left(x^3y'^2-8(x^2-x)yy'+4y^2+8x^2y'\right)dx,\quad y(2)=-7;$

9. $v[y]=\int_1^2\left(\frac{y'^2}{x}+\frac{3y^2}{x^3}\right)dx,\quad y(2)=\frac{19}{2};$

10. $v[y]=\int_{x_0}^{x_1}\sqrt{1+y'^2}dx,\quad y_0=x_0^2+2,\quad y_1=x_1.$

6. (по 3 б.) Найти экстремали в следующих задачах на условный экстремум:

1. $v[y]=\int_0^{\pi}y'^2\,dx,\quad y(0)=0,\quad y(\pi)=\pi,\quad \int_0^{\pi}y\sin x dx=0;$

2. $v[y]=\int\limits_0^1 y'^2\,dx, \quad y(0)=0, \quad y(1)=e-3, \quad \int\limits_0^1 ye^x dx=0;$

3. $v[y]=\int\limits_0^1 y'^2\,dx, \quad y(0)=2e+1, \quad y(1)=2, \quad \int\limits_0^1 ye^{-x} dx=e;$

4. $v[y]=\int\limits_0^1 y'^2\,dx, \quad y(0)=0, \quad y(1)=2, \quad \int\limits_0^1 xydx=1;$

5. $v[y]=\int\limits_0^1 \left(y^2+y'^2\right)dx, \quad y(0)=0, \quad y(1)=-1, \quad \int\limits_0^1 ye^{-x} dx=\frac{3e^{-1}-e}{4};$

6. $v[y]=\int\limits_0^1 \left(y^2+y'^2\right)dx, \quad y(0)=0, \quad y(1)=4e, \quad \int\limits_0^1 ye^{x} dx=1+e^2;$

7. $v[y]=\int\limits_0^{\pi} \left(y'^2+y^2+2y\cos x\right)dx, \quad y(0)=2, \quad y(\pi)=-2, \quad \int\limits_0^{\pi} y\cos xdx=\pi;$

8. $v[y]=\int\limits_1^2 xy'^2\,dx, \quad y(1)=0, \quad y(2)=12, \quad \int\limits_1^2 xydx=9;$

9. $v[y]=\int\limits_0^{\pi} \left(2y+3y'+y'^2\right)dx, \quad y(0)=0, \quad y(\pi)=\pi^2, \quad \int\limits_0^{\pi} y\sin xdx=\pi^2-1;$

10. $v[y]=\int\limits_0^1 \left(2xy+y'^2\right)dx, \quad y(0)=0, \quad y(1)=3, \quad \int\limits_0^1 xydx=1;$

11. $v[y,z]=\int\limits_0^1 \left(y'^2(x)+z'^2(x)\right)dx, \quad y(0)=0, \quad y(1)=1, \quad z(0)=1, \quad z(1)=0, \ \ y'-z=0;$

12. $v[y,z]=\int\limits_0^1 \sqrt{1+y'^2(x)+z'^2(x)}dx, \quad y(0)=1, \quad y(1)=2, \quad z(0)=2, \quad z(1)=1, \quad 2y-z-3x=0;$

13. $v[y,z]=\int\limits_0^{\pi/2} \left(y'^2+z'^2+2yz\right)dx, \quad y(0)=1, \quad y\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}+1, \quad z(0)=-1, \quad z\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}-1, \quad y'+z'-4x=0;$

14. $v[y,z]=\int\limits_0^1 \left(y'^2+z'^2\right)dx, \quad y(0)=-1, \quad y(1)=-1, \quad z(0)=0, \quad z(1)=1, \quad y+z-2x^2+x+1=0;$

15. $v[y,z]=\int\limits_0^{\pi} \left(y'^2+z'^2\right)dx, \quad y(0)=0, \quad y(\pi)=0, \quad z(0)=0, \quad z(\pi)=\frac{\pi}{2}, \quad y'-z-x\cos x=0;$

16. $v[y,z]=\int\limits_0^1 \left(y'^2+z'^2+1\right)dx, \quad y(0)=0, \quad y(1)=2, \quad z(0)=0, \quad z(1)=0, \quad y+z-2x^2=0;$

17. $v[y,z]=\int\limits_0^1 \left(y^2+2y'^2+z'^2\right)dx, \quad y(0)=1, \quad y(1)=e+e^{-1}, \quad z(0)=0, \quad z(1)=2e-e^{-1}, \quad y'(x)-z(x)=0;$

18. $v[y,z]=\int\limits_0^1 \left(y'^2+z'^2+x^3\right)dx, \quad y(0)=2, \quad y(1)=1, \quad z(0)=1, \quad z(1)=2, \quad y-2z+3x=0;$

19. $v[y,z]=\int\limits_0^{\pi/2} \left(y'^2(x)-z'^2(x)\right)dx, \quad y(0)=0, \quad y\left(\frac{\pi}{2}\right)=\frac{\pi}{4}, \quad z(0)=0, \quad z\left(\frac{\pi}{2}\right)=-\frac{1}{2}, \quad y'-z-\sin x=0;$

20. $v[y,z]=\int\limits_0^1 \left(y'^2+2yz+z'^2\right)dx, \quad y(0)=1, \quad y(1)=e, \quad z(0)=1, \quad z(1)=e^{-1}, \quad y-z-e^x+e^{-x}=0.$

Часть 2. Оптимальное управление (предварительный срок сдачи середина июня 2018 г.)

1. (3 б) Проверить на оптимальность управление $u = u(t)$:

1. $\dot{x}_1 = tu, \quad \dot{x}_2 = x_1 u, \quad x_1(0) = x_2(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = -x_2(1) \to \min,$
$u(t) = t, \quad t \in [0,1];$
2. $\dot{x}_1 = x_2 u, \quad \dot{x}_2 = tu, \quad x_1(0) = x_2(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = -x_1(1) \to \min,$
$u(t) \equiv 1, \quad t \in [0,1];$
3. $\dot{x}_1 = ux_2, \quad \dot{x}_2 = u, \quad x_1(0) = 0, \ x_2(0) = 1, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = -2x_1(1) + 3x_2(1) \to \min,$
$u(t) \equiv 1, \quad t \in [0,1];$
4. $\dot{x}_1 = ux_2, \quad \dot{x}_2 = u, \quad x_1(0) = 0, \ x_2(0) = 1, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = -2x_1(1) + 3x_2(1) \to \min,$
$u(t) \equiv 0, \quad t \in [0,1];$
5. $\dot{x}_1 = u, \quad \dot{x}_2 = x_1^2, \quad x_1(0) = 1, \ x_2(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = x_2(1) \to \min,$
$u(t) = t^2, \quad t \in [0,1];$
6. $\dot{x}_1 = u, \quad \dot{x}_2 = x_1^2, \quad x_1(0) = 1, \ x_2(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = x_2(1) \to \min,$
$u(t) \equiv -1, \quad t \in [0,1];$
7. $\dot{x}_1 = x_2 + u^2, \quad \dot{x}_2 = x_1, \quad x_1(0) = x_2(0) = 1, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = x_1(1) + x_2(1) \to \min,$
$$u(t) = \begin{cases} -1, t \in \left[0, \frac{1}{2}\right), \\ 1, t \in \left[\frac{1}{2}, 1\right]; \end{cases}$$
8. $\dot{x}_1 = -x_2 u_1, \quad \dot{x}_2 = x_1 + 2u_1, \quad \dot{x}_3 = u_2, \quad x_1(0) = x_2(0) = x_3(0) = 0, \ u_1^2 + u_2^2 = 1, \quad t \in [0, \pi],$
$J[u] = x_2(\pi) + x_3(\pi) \to \min, \quad u_1(t) \equiv 1, \quad u_2(t) \equiv 0, \quad t \in [0,1];$
9. $\dot{x}_1 = tu, \quad \dot{x}_2 = x_1 u, \quad x_1(0) = x_2(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = -x_2(1) \to \min,$
$u(t) \equiv 1, \quad t \in [0,1];$
10. $\dot{x}_1 = u, \quad \dot{x}_2 = ux_1, \quad x_1(0) = 1, \ x_2(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = 3x_1(1) - 2x_2(1) \to \min,$
$u(t) \equiv 0, \quad t \in [0,1].$

2. (по 3 б.) Применить принцип максимума для решения следующих задач оптимального управления:

1. $\dot{x} = u, \quad x(1) = 5, \ -3 \le u \le 1, \quad t \in [1,2], \quad J[u] = \int_1^2 (x+u)dt \to \min;$
2. $\dot{x} = u, \quad x(0) = 1, \ |u| \le 1, \quad t \in [0,2], \quad J[u] = x(2) + \int_0^2 (x+u^2)dt \to \min;$
3. $\dot{x} = u, \quad x(0) = 1, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = \sin x(1) \to \min;$
4. $\dot{x} = u, \quad x(0) = 1, \ 0 \le u \le \pi, \quad t \in [0,1], \quad J[u] = \cos x(1) \to \min;$
5. $\dot{x} = u^4, \quad x(0) = 0, \ u \in \{-1, 0, 2\}, \quad t \in [0,1], \quad J[u] = \frac{1}{2} x^2(1) \to \min;$

6. $\dot{x} = (2u-1)x, \quad x(0)=1,\ 0\le u\le 1, \quad t\in\left[0,\frac{1}{2}\right], \quad J[u]=\int_0^{\frac{1}{2}}(u-1)x\,dt\to\min;$

7. $\dot{x} = u, \quad x(0)=0,\ u\in\mathbb{R}, \quad t\in\left[0,\frac{\pi}{4}\right], \quad J[u]=\int_0^{\frac{\pi}{4}}(u^2-x^2-6x\sin 2t)\,dt\to\min;$

8. $\dot{x} = u, \quad x(0)=0,\ |u|\le 1, \quad t\in[0,1], \quad J[u]=\frac{1}{2}\int_0^1(x^2+u^2)\,dt\to\min;$

9. $\dot{x} = (2u-1)x, \quad x(0)=1,\ 0\le u\le 1, \quad t\in[0,1], \quad J[u]=\int_0^1(u-1)x\,dt\to\min;$

10. $\dot{x} = 2u, \quad x(0)=1,\ u\in\mathbb{R}, \quad t\in\left[0,\frac{\pi}{2}\right], \quad J[u]=\int_0^{\frac{\pi}{2}}(-x^2+u^2)\,dt\to\min;$

11. $\dot{x}_1 = x_2, \quad \dot{x}_2 = x_1+u, \quad x_1(0)=1,\ x_2(0)=1,\ |u|\le 1, \quad t\in[0,1], \quad J[u]=x_1(1)+x_2(1)\to\min;$

12. $\dot{x}_1 = x_2, \quad \dot{x}_2 = x_1+u^2, \quad x_1(0)=1,\ x_2(0)=1,\ |u|\le 1, \quad t\in[0,1], \quad J[u]=x_1(1)+x_2(1)\to\min;$

13. $\dot{x}_1 = -x_2+u_1^2, \quad \dot{x}_2 = x_1+2u_2, \quad x_1(0)=0,\ x_2(0)=1,\ u_1\in\{-1,0,2\},\ -1\le u_2\le 2, \quad t\in[0,\pi],$

$J[u]=x_2(\pi)+\int_0^{\pi}u_2\,dt\to\min;$

14. $\dot{x}_1 = -x_2, \quad \dot{x}_2 = x_1+2tu, \quad \dot{x}_3 = u, \quad x_1(0)=x_2(0)=x_3(0)=0, \quad -1\le u\le 4, \quad t\in[0,\pi],$
$J[u]=x_2(\pi)+x_3(\pi)\to\min;$

15. $\dot{x}_1 = tu_1-u_2, \quad \dot{x}_2 = x_1+u_2-2u_3^2, \quad x_1(0)=0,\ x_2(0)=0,\ 0\le u_1\le 2,\ |u_2|\le 1,$
$u_3\in\{-4,0,1,3\}, \quad t\in[0,2], \quad J[u]=x_2(2)\to\min;$

16. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad x_1(0)=0,\ x_2(0)=0,\ |u|\le 1, \quad t\in[0,1], \quad J[u]=x_1^2(1)\to\min;$

17. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad x_1(0)=-1,\ x_2(0)=-1,\ |u|\le 1, \quad t\in[0,1], \quad J[u]=x_1^2(1)\to\min;$

18. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad x_1(0)=1,\ x_2(0)=2,\ |u|\le 1, \quad t\in[0,1], \quad J[u]=x_1^2(1)\to\min;$

19. $\dot{x}_1 = u_1-u_2, \quad \dot{x}_2 = x_1+2u_2+u_3, \quad x_1(0)=0,\ x_2(0)=0,\ u_1^2+u_3^2\le 1, |u_2|=1, \quad t\in[0,1],$
$J[u]=x_2(1)\to\min;$

20. $\dot{x}_1 = x_2+2u_2, \quad \dot{x}_2 = -x_1+u_1^2, \quad x_1(0)=1,\ x_2(0)=0,\ u_1\in\{-1,0,2\},\ -1\le u_2\le 2, \quad t\in[0,\pi],$

$J[u]=x_1(\pi)+\int_0^{\pi}u_2\,dt\to\min.$

3. (3 б.) Провести одну итерацию метода игольчатой вариации:

1. $\dot{x} = u, \quad x(0)=0, \quad |u|\le 1, \quad t\in[0,3], \quad J[u]=-4x(3)-\frac{1}{2}x^2(3)+\int_0^3 u(4x-t)\,dt\to\min,$

$u^0(t)\equiv 0;$

2. $\dot{x} = u, \quad x(0)=1, \quad -3\le u\le -1, \quad t\in[0,1], \quad J[u]=-5x(1)+\int_0^1 x(u+1)\,dt\to\min,$

$u^0(t)\equiv -2;$

3. $\dot{x}_1 = -6x_2, \quad \dot{x}_2 = -u, \quad \dot{x}_3 = ux_2, \quad x_1(0)=0,\ x_2(0)=7,\ x_3(0)=0,\ |u|\le 1, \quad t\in[0,3],$
$J[u]=x_1(3)+4x_2(3)+2x_3(3)\to\min, \quad u^0(t)\equiv 1;$

4. $\dot{x}_1 = -u, \quad \dot{x}_2 = x_1, \quad x_1(0) = -5, \ x_2(0) = 0, \ \ |u| \le 2, \quad t \in [0,4],$

$J[u] = 8x_1(4) + 3x_2(4) - 4\int\limits_0^4 x_1 u dt \to \min, \quad u^0(t) \equiv 1;$

5. $\dot{x} = 2u, \quad x(0) = 1, \ \ 0 \le u \le 2, \quad t \in [0,2], \quad J[u] = -2x(2) - \int\limits_0^2 x(u-2)dt \to \min,$

$u^0(t) \equiv 1;$

6. $\dot{x}_1 = (x_2 + 1)u, \quad \dot{x}_2 = 2u - 1, \quad x_1(0) = 0, \ x_2(0) = 1, \ \ 0 \le u \le 2, \quad t \in [0,3],$

$J[u] = -x_1(3) + 4x_2(3) \to \min, \quad u^0(t) \equiv 1;$

7. $\dot{x}_1 = u, \quad \dot{x}_2 = x_1, \quad x_1(0) = 0, \ x_2(0) = -1, \ |u| \le 1, \quad t \in [0,3], \quad J[u] = x_1^2(3) + x_2(3) \to \min,$

$u^0(t) \equiv 0;$

8. $\dot{x} = u, \quad x(0) = 0, \ \ |u| \le 1, \quad t \in [0,2], \quad J[u] = \int\limits_0^2 \left(\frac{1}{2}x^2 + 2(t-2)u \right) dt \to \min,$

$u^0(t) \equiv 0;$

9. $\dot{x} = u, \quad x(0) = 1, \ \ |u| \le 1, \quad t \in [0,1], \quad J[u] = \int\limits_0^1 x(u-1)dt \to \min,$

$u^0(t) \equiv 0;$

10. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad \dot{x}_3 = ux_1, \quad x_1(0) = 0, \ x_2(0) = 0, \ x_3(0) = 0, \ |u| \le 1, \quad t \in [0,1],$

$J[u] = x_3(1) \to \min, \quad u^0(t) \equiv 1.$

4. (3 б.) Провести одну итерацию метода условного градиента:

1. $\dot{x}_1 = u, \quad \dot{x}_2 = 2x_1 u, \quad x_1(0) = 0, \ x_2(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = x_1(1) + x_2(1) \to \min,$

$u^0(t) \equiv 0;$

2. $\dot{x}_1 = 4u, \quad \dot{x}_2 = \frac{1}{4}x_1, \quad x_1(0) = 4, \ x_2(0) = 0, \ |u| \le 1, \quad t \in [0,4],$

$J[u] = -3x_1(4) + x_2(4) + \int\limits_0^4 x_1 u dt \to \min, \quad u^0(t) \equiv -\frac{1}{4};$

3. $\dot{x} = 2u, \quad x(0) = 0, \ |u| \le 1, \quad t \in [0,1], \quad J[u] = \frac{1}{2}\int\limits_0^1 (x^2 + 2u^2)dt \to \min, \quad u^0(t) \equiv 1;$

4. $\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u, \quad x_1(0) = 1, \ x_2(0) = 0, \ |u| \le 1, \quad t \in \left[0, \frac{\pi}{2}\right],$

$J[u] = \frac{1}{2}x_1^2\left(\frac{\pi}{2}\right) + \frac{1}{2}x_2^2\left(\frac{\pi}{2}\right) \to \min, \quad u^0(t) \equiv 0;$

5. $\dot{x}_1 = 3u, \quad \dot{x}_2 = x_1 u, \quad x_1(0) = 0, \ x_2(0) = 0, \ |u| \le 2, \quad t \in [0,4],$

$J[u] = -6x_1(4) + x_2(4) + 2\int\limits_0^4 x_1 dt \to \min, \quad u^0(t) \equiv -1;$

6. $\dot{x} = u, \quad x(0) = 0, \ 1 \le u \le 3, \quad t \in [0,3], \quad J[u] = 4x(3) + 3\int\limits_0^3 x(u+1)dt \to \min, \quad u^0(t) \equiv 2;$

7. $\dot{x} = 2u, \quad x(0) = 0, \ -1 \le u \le 3, \quad t \in [0,2], \quad J[u] = 4x(2) + \int\limits_0^2 x(u-2)dt \to \min, \quad u^0(t) \equiv 1;$

8. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad \dot{x}_3 = ux_1, \quad x_1(0) = 0, \ x_2(0) = 0, \ x_3(0) = 0, \quad |u| \le 1, \quad t \in [0,1],$

$J[u] = x_3(1) \to \min, \quad u^0(t) \equiv 1.$

9. $\dot{x}_1 = u, \quad \dot{x}_2 = x_1^2, \quad \dot{x}_3 = (t-2)u, \quad x_1(0)=0,\ x_2(0)=0,\ x_3(0)=0, \quad -1 \le u \le 2, \quad t \in [0,1],$
$J[u] = \frac{1}{2} x_2(1) + 2x_3(1) \to \min, \quad u^0(t) \equiv 0;$

10. $\dot{x} = u, \quad x(0)=1,\ |u| \le 1, \quad t \in [0,1], \quad J[u] = \int_0^1 x(u-1)dt \to \min, \quad u^0(t) \equiv 1.$

5. (3 б.) Провести одну итерацию метода проекции градиента:

1. $\dot{x} = u, \quad x(0)=0,\ |u| \le 1, \quad t \in [0,2], \quad J[u] = \int_0^2 \left(\frac{1}{2} x^2 - 2(t+2)u \right) dt \to \min, \quad u^0(t) \equiv 0;$

2. $\dot{x}_1 = -3u, \quad \dot{x}_2 = 2x_1 u, \quad \dot{x}_3 = 6x_1, \quad x_1(0)=0,\ x_2(0)=0,\ x_3(0)=0,$
$-3 \le u \le 1, \quad t \in [0,3], \quad J[u] = 4x_1(3) - x_2(3) + x_3(3) \to \min, \quad u^0(t) \equiv -2;$

3. $\dot{x} = -u, \quad x(0)=0,\ 2 \le u \le 4, \quad t \in [0,3], \quad J[u] = 4x(3) - \int_0^3 x(u+1)dt \to \min, \quad u^0(t) \equiv 3;$

4. $\dot{x}_1 = -u, \quad \dot{x}_2 = 4x_1, \quad x_1(0)=0,\ x_2(0)=0,\ |u| \le 3, \quad t \in [0,2],$
$J[u] = -2x_1(2) + x_2(2) - 4\int_0^2 x_1 u dt \to \min, \quad u^0(t) \equiv -1;$

5. $\dot{x} = u+1, \quad x(0)=-1,\ -2 \le u \le 0, \quad t \in [0,2], \quad J[u] = \frac{3}{2} x^2(2) - \int_0^2 xu dt \to \min,$
$u^0(t) \equiv -1;$

6. $\dot{x} = -u, \quad x(0)=0,\ |u| \le 2, \quad t \in [0,2], \quad J[u] = x(2) + 2\int_0^2 x(u+1)dt \to \min, \quad u^0(t) \equiv 1;$

7. $\dot{x}_1 = x_2 u, \quad \dot{x}_2 = 2u, \quad x_1(0)=0,\ x_2(0)=0,\ |u| \le 1, \quad t \in [0,4],$
$J[u] = 3x_1(4) - x_2(4) + 6\int_0^4 x_2 u dt \to \min, \quad u^0(t) \equiv 1;$

8. $\dot{x}_1 = -3u, \quad \dot{x}_2 = 6x_1, \quad \dot{x}_3 = 2x_1 u, \quad x_1(0)=0,\ x_2(0)=0,\ x_3(0)=0,$
$-3 \le u \le 1, \quad t \in [0,3], \quad J[u] = 4x_1(3) + x_2(3) - x_3(3) \to \min, \quad u^0(t) \equiv -2;$

9. $\dot{x}_1 = 2u, \quad \dot{x}_2 = x_1 u, \quad x_1(0)=0,\ x_2(0)=0,\ |u| \le 1, \quad t \in [0,4],$
$J[u] = -x_1(4) + 3x_2(4) + 6\int_0^4 x_1 u dt \to \min, \quad u^0(t) \equiv 1;$

10. $\dot{x}_1 = 4x_2, \quad \dot{x}_2 = -u, \quad x_1(0)=0,\ x_2(0)=0,\ |u| \le 3, \quad t \in [0,2],$
$J[u] = x_1(2) - 2x_2(2) - 4\int_0^2 x_2 u dt \to \min, \quad u^0(t) \equiv -1.$

6. (3 б.) Решить задачу синтеза:

1. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad |u| \le \frac{3}{2};$
2. $\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u, \quad 1 \le u \le 2;$
3. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad 0 \le u \le 1;$
4. $\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u, \quad 0 \le u \le 1;$
5. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad -1 \le u \le 0;$
6. $\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u, \quad -1 \le u \le 0;$

7. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad 1 \le u \le 2;$

8. $\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u, \quad |u| \le 2;$

9. $\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad -2 \le u \le -1;$

10. $\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u, \quad -2 \le u \le 1.$